Diffraction Grating

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B.Sc Part-II
Paper-III

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Diffraction Grating-Normal incidence–(Diffraction at N parallel slits)

Construction

An arrangement consisting of large number of parallel slits of the same width and separated by equal opaque spaces is known as diffraction grating. Fraunhofer used the first grating consisting of a large number of parallel wires placed very closely side by side at regular intervals. The diameter of the wires was of the order of 0.05mm and their spacing varied from0.0533 mm to 0.687 mm. Now gratings are constructed by ruling equidistant parallel lines on a transparent material such as glass with a fine diamond point. The ruled lines are opaque to light while the space between any two lines is transparent to light and acts as a slit. This is known as *Plane transmission grating.* On the other hand, if the lines are drawn on a silvered surface (plane or concave) then the light is reflected from the positions of mirrors in between any two lines and it forms a *plane or concave reflection grating*. When the spacing between the lines is of the order of the wavelength of light, then an appreciable deviation of light is produced.

Theory

Fig. represents the section of a plane transmission grating placed perpendicular to the plane of the paper. Let ‘e’ be the width of each slit and‘d’ be the width of each opaque part. Then (e+d) is known as grating element. XY is the screen placed perpendicular to the plane of a paper. Suppose a parallel beam of monochromatic light of wavelength λ be incident normally on the grating. By Huygen's principle, each of the slit sends secondary wavelets in all directions. The secondary wavelets travelling in the same direction of incident light will come to a focus at a point P_0 of the screen as the screen is placed at the focal plane of the convex lens. The point P_0 will be the central maximum. Now, consider the secondary waves travelling in a direction inclined at an angle θ with the direction of the incident light. These waves reach the point P_1 on passing through the convex lens in different phases. As a result, dark and bright bands on both sides of the central maximum are obtained.

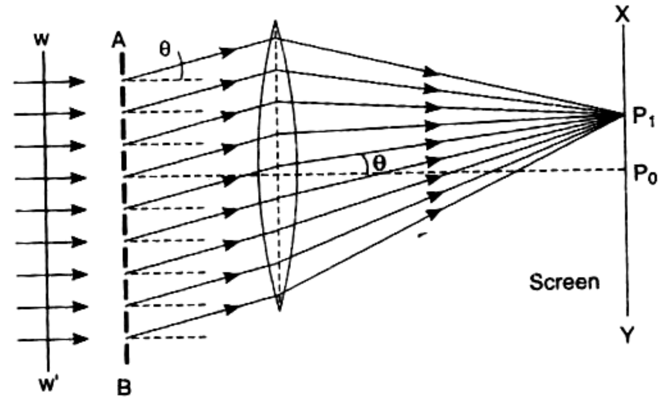


Fig. *Section of a Plane transmission grating*

The intensity at point P_1 may be considered by applying the theory of Fraunhofer diffraction at a single slit. The wavelets proceeding from all points in a slit along the direction ϴ are equivalent to a single wave of amplitude (A $\sin\alpha/\alpha$) starting from the middle point of the slit, where

$$\alpha = (\pi\, e \sin\theta/\lambda).$$

If there are N slits, then there will be N diffracted waves, one each from the middle points of the slits. The path difference between two consecutive slits is $(e+d)\sin\theta$. Therefore, there is a corresponding phase difference of $(2\pi/\lambda)$. $(e+d)\sin\theta$ between the two consecutive waves. The phase difference is constant and it is 2β.

Hence, the problem of determining the intensity in the direction ϴ reduces to finding the resultant amplitude of N vibrations each of amplitude ($A\sin\propto/\propto$) and having a common phase difference

$$\frac{2\pi}{\lambda}(e+d)\sin\theta = 2\beta \qquad \rightarrow (1)$$

Now, by the method of vector addition of amplitudes, the direction of ϴ will be

$$R^* = \frac{A\sin\propto}{\propto} \cdot \frac{\sin N\beta}{\sin\beta}$$

$$\text{And } I = R^2 = \left(\frac{A\sin\propto}{\propto}\right)^2 \left(\frac{\sin N\beta}{\sin\beta}\right)^2 = I_o \left(\frac{\sin\propto}{\propto}\right)^2 \left(\frac{\sin^2 N\beta}{\sin^2\beta}\right) \qquad \rightarrow (2)$$

The factor $\left(\frac{A\sin\propto}{\propto}\right)^2$ gives the distribution of intensity due to single slit while the factor $(\sin^2 N\beta/\sin^2\beta)$ gives the distribution of intensity as a combined effect of all the slits.

Intensity distribution in N-Slits

Principle maxima

The intensity would be maximum when $\sin\beta = 0$.

or $\beta = \pm n\pi$ *where*, n=0, 1, 2, 3,…

but at the same time $\sin N\beta = 0$, so that the factor ($\sin N\beta/\sin\beta$) becomes indeterminate. It may be evaluated by applying the usual method of differentiating the numerator and the denominator, i.e., by applying the Hospital's rule. Thus,

$$\lim_{\beta \to \pm n\pi} \frac{\sin N\beta}{\sin\beta}$$

$$= \lim_{\beta \to \pm n\pi} \left(\frac{\frac{d}{d\beta}(\sin N\beta)}{\frac{d}{d\beta}(\sin\beta)} \right)$$

$$\lim_{\beta \to \pm n\pi} \left(\frac{N \cos N\beta}{\cos\beta} \right) = \pm N$$

$$\text{Hence, } \lim_{\beta \to \pm n\pi} \left(\frac{\sin N\beta}{\sin\beta} \right)^2 = N^2$$

The resultant intensity is $\left(\frac{A \sin\alpha}{\alpha}\right) N^2$. The maxima are most intense and are called as principal maxima.

The maxima are obtained for

$$\beta = \pm n\pi$$

$$\frac{\pi}{\lambda} (e+d) \sin\Theta = \pm n\pi$$

$$or\ (e+d)\sin\theta = \pm n\lambda \quad \text{Where, n= 0, 1, 2, 3….}$$

$n = 0$ corresponds to zero order maximum. For n=1, 2, 3...etc., the first, second, third, etc., principal maxima are obtained respectively. The $\pm$ sign shows that there are two principal maxima of the same order lying on the either side of zero order maximum.

Minima

A series of minima occur, when

$$\sin N\beta = 0 \qquad\qquad \text{but } \sin\beta \neq 0$$

For minima,

$$\sin N\beta = 0$$

$$N\beta = \pm m\pi$$

$$N\frac{\pi}{\lambda} (e+d) \sin\Theta = \pm m\pi$$

$$N (e+d) \sin\Theta = \pm m\lambda$$

Where m has all integral values except 0, N, 2N …nN, because for these values sinβ becomes zero and the principal maxima is obtained. Thus, m = 1, 2, 3… (N-1). Hence, they are adjacent principal maxima.

Secondary maxima

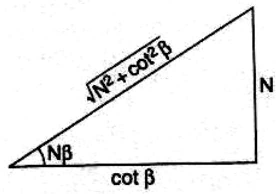
As there are (N-1) minima between two adjacent principal maxima, there must be (N-2) other maxima between two principal maxima. To find out the position of these secondary maxima, equation (2) must be differentiated with respect to β and then equate it to zero. Thus,

$$\frac{dI}{d\beta} = \left(\frac{A \sin\alpha}{\alpha}\right)^2 . 2\left(\frac{\sin N\beta}{\sin\beta}\right)$$

$$\left[\frac{N \cos N\beta \sin\beta - \sin N\beta \cos\beta}{\sin^2\beta}\right] = 0 \text{ or}$$

$$N \cos N\beta \sin\beta - \sin N\beta \cos\beta = 0$$
$$N \tan\beta = \tan N\beta$$

The roots of this equation other than those for which $\beta = \pm n\pi$ (which correspond to principal maxima) give the positions of secondary maxima. To find out the value of ($\sin^2 N\beta/\sin^2\beta$) from equation N $\tan\beta = \tan N\beta$, a triangle shown below, in the figure [redacted] is used.



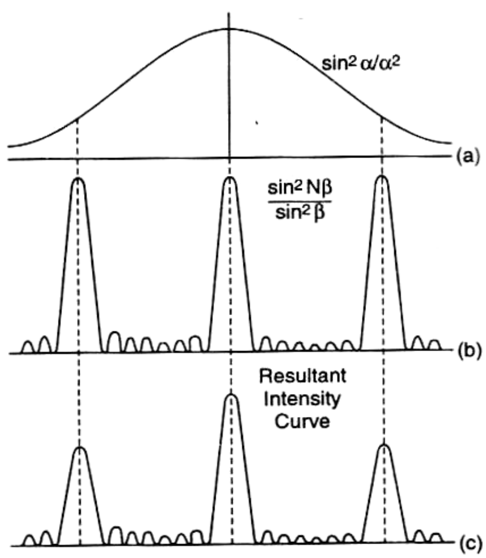
Fig

$$\text{Sin } N\beta = \frac{N}{\sqrt{(N^2+\cot^2\beta)}}$$
$$\frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2}{(N^2+\cot^2\beta)\times\sin^2\beta}$$
$$= \frac{N^2}{N^2\sin^2\beta+\cos^2\beta}$$
$$= \frac{N^2}{1+(N^2-1)\sin^2\beta}$$

Therefore, $\frac{Intensity\ \ of\ secondary\ \ maxima}{Intensity\ \ of\ principa\ l\ maxima} = \frac{1}{1+(N^2-1)\sin^2\beta}$

As N increases, the intensity of secondary maxima relative to principal maxima decreases and becomes negligible when N becomes large.

Figures 7.16 (a) and (b) show the graphs of variation of intensity due to the factors $sin^2 \propto/\propto^2$ and $sin^2 N\beta/sin^2\beta$ respectively. The resultant is shown in figure (c).



(a) & (b) Graphs showing the variation of intensity (c) The Resultant

Diffraction Grating

An arrangement which consists of a large number of parallel slits of the same width and separated by equal opaque spaces is known as diffraction grating. Fraunhofer used the first grating consisting of a large number of parallel wires placed side by side very closely at regular intervals. Now gratings are constructed by ruling equidistant parallel lines on a transparent

material such as glass with a fine diamond point. The ruled lines are opaque to light while the space between any two lines is transparent to light and acts as a slit .
Commercial gratings are produced by taking the cast of an actual grating on a transparent film like that of cellulose acetate. Solution of cellulose acetate is poured on the ruled surface and allowed to dry to form a thin film, detachable from the surface. These impressions of a grating are preserved by mounting the film between two glass sheets.

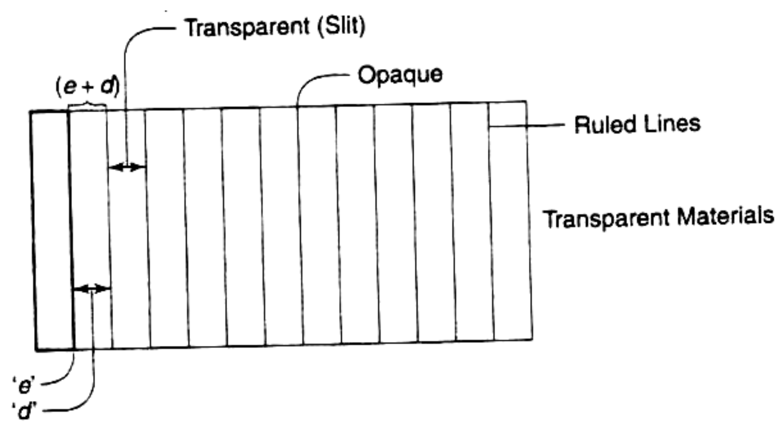


Fig. Diffraction Grating

Let 'e' be the width of the line and'd' be the width of the slit. Then (e+d) is known as *grating element.* If 'N' is the number of lines per inch on the grating, then

$$N(e+d) = 1'' = 2.54 \text{ cm}$$
$$e+d = \frac{2.54}{N} \text{ cm}$$

There will be nearly 30,000 lines per inch of a grating. Due to the above fact, the width of the slit is very narrow and is comparable to the wavelength of light. When light falls on the grating, the light gets diffracted through each slit. As a result, both diffraction and interference of diffracted light gets enhanced and forms a diffraction pattern. This pattern is known as *diffraction spectrum.*

Grating Spectrum

The condition to form the principal maxima in a grating is given by
$(e+d) \sin \Theta = n\lambda$
Where (e+d) is the grating element and the above equation is known as grating equation.
From the grating equation, the following is clear.

1. For a particular wavelength λ, the angle of diffraction Θ is different for principal maxima of different orders.
2. As the number of lines in the grating is large, maxima appear as sharp, bright parallel lines and are termed as spectral lines.
3. For white light and for a particular order of n, the light of different wavelengths will be diffracted in different directions.
4. At the center, $\Theta=0$ gives the maxima of all wavelengths which coincides to form the central image of the same colour as that of the light source. This forms zero order.( )

5. The principal maxima of all wavelengths forms the first, second,… order spectra for n=1,2,…
6. The longer the wavelength, greater is the angle of diffraction. Thus, the spectrum consists of violet being in the innermost position and red being in the outermost positions.

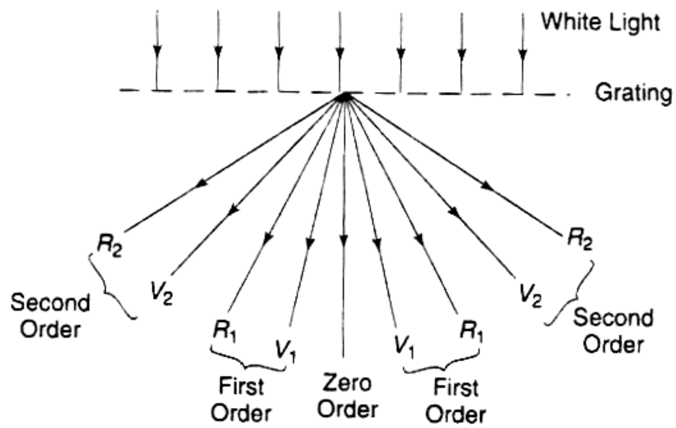


Fig: Grating Spectrum

7. Most of the intensity goes to zero order and the rest is distributed among other orders.
8. Spectra of different orders are situated symmetrically on both sides of zero order.
9. The maximum number of orders available with the grating is $n_{max} = \frac{(e+d)}{\lambda}$